DETERMINING THE TOP-ANTITOP AND ZZ COUPLINGS OF A NEUTRAL HIGGS BOSON OF ARBITRARY CP NATURE AT THE NLC

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Abstract

The optimal procedure for extracting the coefficients of different components of a cross section which takes the form of unknown coefficients times functions of known kinematical form is developed. When applied to $e^+e^- \to t\bar{t}$+Higgs production at $\sqrt{s} = 1\,\mathrm{TeV}$ and integrated luminosity of $200\ \mathrm{fb}^{-1}$, we find that the $t\bar{t} \to$Higgs CP-even and CP-odd couplings and, to a lesser extent, the $ZZ \to$Higgs (CP-even) coupling can be extracted with reasonable errors, assuming the Higgs sector parameter choices yield a significant production rate. Indeed, the composition of a mixed-CP Higgs eigenstate can be determined with sufficient accuracy that a SM-like CP-even Higgs boson can be distinguished from a purely CP-odd Higgs boson at a high level of statistical significance, and vice versa.

1 Introduction

If Higgs boson(s) (generically denoted as h) exist and are discovered at either the CERN LHC or a future next linear e^+e^- collider (NLC), it will be extremely important to determine both the magnitude and the CP nature of their couplings. As reviewed in Ref. [1], determining the CP properties of a Higgs boson through its couplings will be especially challenging. The most promising approaches proposed to date include: photon polarization asymmetries in $\gamma\gamma \to h$ [2]; momentum correlations among the final state τ or t decay products appearing in $e^+e^- \to Zh$ and $\mu^+\mu^- \to h$ with $h \to \tau^+\tau^-$ or $t\bar{t}$, respectively [3, 4]; and weighted cross section integrals in $pp \to t\bar{t}h$ at the LHC [5] and in $e^+e^- \to t\bar{t}h$ at the NLC [6]. The latter $t\bar{t}h$ analyzes examined what can be accomplished using a single observable. Ref. [5] found that under ideal circumstances a SM-like CP-even Higgs boson can be distinguished from a purely CP-odd Higgs boson at a statistically significant level using $pp \to t\bar{t}h$ data from the LHC. Ref. [6] found that a statistically significant signal for the CP-violating cross term generically present in the $e^+e^- \to t\bar{t}h$ cross section

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for a Higgs boson with both CP-even and CP-odd components might be possible. However, neither of these analyzes took full advantage of all the information available in the cross section as a function of the kinematical variables.

In this letter, we outline the optimal technique for determining the coefficients c_i appearing in a cross section that can be written in the generic form $d\sigma/d\phi = \sum_i c_i f_i(\phi)$, where ϕ denotes the final state phase space configuration. The application upon which we shall focus is $e^+e^- \to t\bar{t}h$ production, where the c_i are functions of the Higgs couplings. By extracting the c_i we can determine all the Higgs couplings and, thence, its CP nature. This use of the full information contained in the final state distributions, as encoded in the c_i, leads to significant improvement in the statistical precision with which the couplings/CP-nature of a Higgs boson can be determined. For example, in $e^+e^- \to t\bar{t}h$ at $\sqrt{s} = 1\,\text{TeV}$, if $L = 200\ \text{fb}^{-1}$ and final state reconstruction efficiency is of order $\epsilon = 0.25$, a SM-like CP-even Higgs boson can be distinguished from a pure CP-odd Higgs boson at roughly the 9.5σ statistical level.

2 General Technique

We assume that

$$\Sigma(\phi) \equiv \frac{d\sigma}{d\phi} = \sum_i c_i f_i(\phi)\,, \tag{1}$$

where the $f_i(\phi)$ are known functions of the location in final state phase space, ϕ, and the c_i are model-dependent coefficients (taken to be dimensionless in our convention). The coefficients c_i can be extracted by using appropriate weighting functions $w_i(\phi)$ such that $\int w_i(\phi)\Sigma(\phi) = c_i$. In general, different choices for the $w_i(\phi)$ are possible. However, there is a unique choice such that the statistical error in the determination of the c_i is minimized in the sense that the entire covariance matrix is at a stationary point in terms of varying the functional forms for the $w_i(\phi)$ while maintaining $\int w_i(\phi) f_j(\phi) d\phi = \delta_{ij}$. Thus, we require

$$(a): \quad \delta V_{ij} \propto \int \delta\left[w_i(\phi) w_j(\phi)\right] \Sigma(\phi) d\phi = 0\,, \quad (b): \quad \int \delta w_i(\phi) f_j(\phi) = 0\,, \tag{2}$$

where V_{ij} is the covariance matrix. The weighting functions which satisfy these conditions are of the form

$$w_i(\phi) = \frac{\sum_j X_{ij} f_j(\phi)}{\Sigma(\phi)}\,, \quad \text{with} \ \ X_{ij} = M_{ij}^{-1}\,, \quad \text{where} \ \ M_{ik} \equiv \int \frac{f_i(\phi) f_k(\phi)}{\Sigma(\phi)} d\phi\,, \tag{3}$$

since, for the $w_i(\phi)$ so defined, the constraint (b) implies the minimization condition (a) in Eq. (2).

We may then compute c_i as

$$c_i = \sum_k X_{ik} I_k = \sum_k M_{ik}^{-1} I_k\,, \quad \text{where} \ \ I_k \equiv \int f_k(\phi) d\phi\,. \tag{4}$$

It can then be demonstrated that the covariance matrix is

$$V_{ij} \equiv \langle \Delta c_i \Delta c_j \rangle = \frac{M_{ij}^{-1} \sigma_T}{N} \,, \tag{5}$$

where $\sigma_T = \int \frac{d\sigma}{d\phi} d\phi$ is the integrated cross section and $N = L_{\text{eff}} \sigma_T$ is the total number of events, with L_{eff} being the luminosity times efficiency. The result of Eq. (5) applies only for the optimal weighting functions.

We note that the above procedure is the optimal one regardless of the relative magnitudes of the c_i. Various limits of the optimal weighting functions have previously appeared in the literature. For example, if all of the c_i are small except for c_k, then to isolate the c_i ($i \neq k$) the appropriate weighting function reduces to $w_i(\phi) \propto f_i(\phi)/f_k(\phi)$, see *e.g.* Refs. [3, 4, 8].

Our procedure is not altered if cuts are imposed on the kinematical phase space over which one integrates. Although such cuts may be required in the actual experimental analysis, we have not included cuts in our model computations to follow.

In the $e^+e^- \rightarrow t\bar{t}h$ process, upon which we shall now focus, in order to fully define a point in phase space we must identify the t and $\bar{t}$ and have no more than one invisible particle. Thus, we must employ the final state mode in which one t decays leptonically and the other hadronically. Further, the t and $\bar{t}$ must be reconstructed. The overall efficiency for the mixed leptonic-hadronic final state decays and the double reconstruction will be denoted by ϵ. Then the effective luminosity is given by $L_{\text{eff}} = \epsilon L$, where L is the total integrated luminosity. We shall take $\epsilon = 0.25$.

If a subset, $\bar{\phi}$, of the kinematical variables ϕ cannot be determined (as is the case for $e^+e^- \rightarrow t\bar{t}h$ if the $t,\bar{t}$ decay to the purely hadronic or double-leptonic final state) the above technique can be applied using the variables, $\hat{\phi}$, that *can* be observed and the functions $\hat{f}_i(\hat{\phi}) \equiv \int f_i(\phi) d\bar{\phi}$. This is the case even if one or more $\hat{f}_k$ are zero. For example, in $e^+e^- \rightarrow t\bar{t}h$ the $f_k(\phi)$ that is a CP-odd function of the variables ϕ reduces to $\hat{f}_k = 0$ if one cannot distinguish between the t and $\bar{t}$.

3   Extracting Higgs Couplings in $e^+e^- \rightarrow t\bar{t}h$

We have applied the above procedure to the extraction of Higgs couplings using the process $e^+e^- \rightarrow t\bar{t}h$. We define the Higgs couplings via the Feynman rules:

$$t\bar{t}h : \quad -\bar{t}(a + ib\gamma_5)t\frac{gm_t}{2m_W} \,, \quad ZZh : \quad c\frac{gm_Z}{\cos(\theta_W)}g_{\mu\nu} \,, \tag{6}$$

where g is the usual electroweak coupling constant. Thus, a, b, and c are defined relative to couplings of SM-magnitude. The SM Higgs boson has $a = c = 1$ and $b = 0$. A purely CP-odd Higgs boson has $a = c = 0$ and $b \neq 0$; the magnitude of b depends upon the model — we will display results for $b = 1$, which would

correspond to $\tan\beta = 1$ in a two-Higgs-doublet model of type II (see Refs. [1, 7] for details). This latter choice would, in particular, apply for the CP-odd A^0 of the minimal supersymmetric model (with $\tan\beta = 1$).

The $t\bar{t}h$ cross section contains five distinct terms: $\Sigma(\phi) = \sum_{i=1}^{5} c_i f_i(\phi)$, where

$$c_1 = a^2 \,; \quad c_2 = b^2 \,; \quad c_3 = c^2 \,; \quad c_4 = ac \,; \quad c_5 = bc \,. \tag{7}$$

Of these, the only term in $\Sigma(\phi)$ that is actually CP-violating is that proportional to bc; this is the term upon which Ref. [6] focused. Our approach makes use of the fact that the full cross section contains additional information regarding both b and c.

We have considered three distinct Higgs coupling cases:

- I) The Standard Model Higgs boson, with $a = c = 1$, $b = 0$.

- II) A pure CP-odd Higgs boson, with $a = c = 0$, $b = 1$.

- III) A CP-mixed Higgs boson, with $a = b = c = 1/\sqrt{2}$.

For unpolarized beams, $\sqrt{s} = 1\,\mathrm{TeV}$, $m_h = 100\,\mathrm{GeV}$ and $m_t = 176\,\mathrm{GeV}$, the integrated cross sections in cases I, II and III are $\sigma_T = 2.71$, 0.53, and 1.62 fb, respectively. Adopting $L_{\mathrm{eff}} = 50\,\mathrm{fb}^{-1}$, we then computed

$$\chi^2 = \sum_{i,j=1}^{5} (c_i - c_i^0)(c_j - c_j^0)V_{ij}^{-1}\,, \quad \text{with} \ \ V_{ij}^{-1} = \frac{M_{ij}N}{\sigma_T}\,, \tag{8}$$

(see Eq. (5)) as a function of location in a, b, c parameter space, where the c_i^0 for a given case are computed from the model input values of a, b, c (given above) using Eq. (7). Surfaces of constant $\chi^2 = 1$ and 36 are displayed in Fig. 1 for each of the three cases. We have indicated the parameter space location of models I, II and III by a solid bullet, square, and star, respectively. The $\chi^2 = 1$ surfaces indicate the 1σ errors on the parameter determinations. The $\chi^2 = 36$ (or 6σ) surfaces will be useful as a reference in assessing the level at which we can distinguish the above three model cases from one another.

Due to the fact that the five c_i are functions of only the three parameters, a, b, c, the $\chi^2 = 1$ surfaces in Fig. 1 are not perfect ellipsoids. Nonetheless, we follow the usual procedure of defining the $\pm 1\sigma$ errors in any one of the a, b, c parameters by the largest and smallest values that the given parameter takes as one moves about the $\chi^2 = 1$ surface. (These extrema define the locations of the two planes of constant parameter value that are tangent to the $\chi^2 = 1$ surface.) The resulting 1σ errors are tabulated in Table 1. (The upper and lower limits for a, b, c employed for the $\chi^2 = 1$ surface plots of Fig. 1 are only *just* beyond the extrema values.) We observe that a is well determined in all cases, but especially for the $a \neq 0$ cases I and III. Similarly, b is well determined in the $b \neq 0$ cases II and III. The magnitude of the error in c is similar for all three cases, and is never especially
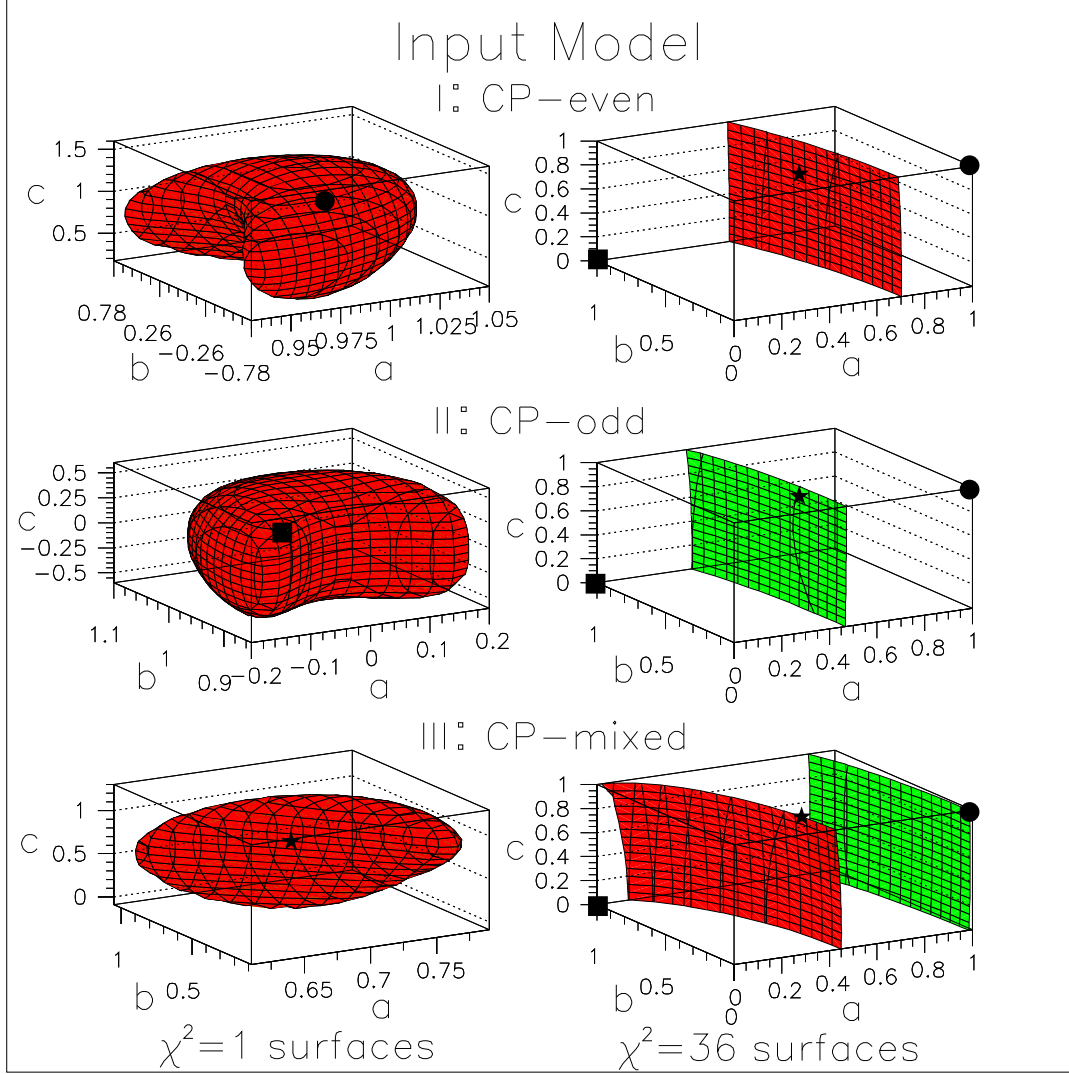
Figure 1: Surfaces of constant $\chi^2 = 1$ and 36 are displayed for: I) $a = c = 1$, $b = 0$; II) $a = c = 0$, $b = 1$; and III) $a = b = c = 1/\sqrt{2}$. The parameter space locations for I, II and III are indicated by a solid bullet, square, and star, respectively. Results are for unpolarized beams, $\sqrt{s} = 1\,\mathrm{TeV}$, $m_h = 100\,\mathrm{GeV}$, $m_t = 176\,\mathrm{GeV}$ and $L_{\mathrm{eff}} = 50\ \mathrm{fb}^{-1}$.

small. Of course, a much better measurement (*e.g.* $\pm 5\%$ for a SM-like h) of or bound on c will be available from inclusive Zh production; however, this does not lead to reduced errors for a and b. Some improvement in the errors is possible if the electron beam can be negatively polarized without loss of luminosity; the errors for $P(e^-) = -1$ are given in the table. In what follows, we shall only consider the case of unpolarized beams.

Table 1: We tabulate the 1σ errors, as defined in the text, in a, b, and c for the three Higgs coupling cases I, II and III, assuming $\sqrt{s} = 1\,\mathrm{TeV}$, $m_h = 100\,\mathrm{GeV}$, $m_t = 176\,\mathrm{GeV}$ and $L_{\mathrm{eff}} = 50\,\mathrm{fb}^{-1}$. Results for unpolarized beams and for 100% negative e^- polarization are given.

Case	Unpolarized e^-			$P(e^-) = -1$		
	$a \pm \Delta a$	$b \pm \Delta b$	$c \pm \Delta c$	$a \pm \Delta a$	$b \pm \Delta b$	$c \pm \Delta c$
I	$1^{+0.043}_{-0.066}$	$0^{+0.76}_{-0.76}$	$1^{+0.51}_{-0.82}$	$1^{+0.037}_{-0.057}$	$0^{+0.70}_{-0.70}$	$1^{+0.51}_{-0.77}$
II	$0^{+0.19}_{-0.19}$	$1^{+0.093}_{-0.14}$	$0^{+0.58}_{-0.58}$	$0^{+0.18}_{-0.18}$	$1^{+0.079}_{-0.12}$	$0^{+0.55}_{-0.55}$
III	$\frac{1}{\sqrt{2}}^{+0.075}_{-0.087}$	$\frac{1}{\sqrt{2}}^{+0.31}_{-0.62}$	$\frac{1}{\sqrt{2}}^{+0.57}_{-0.80}$	$\frac{1}{\sqrt{2}}^{+0.066}_{-0.074}$	$\frac{1}{\sqrt{2}}^{+0.27}_{-0.46}$	$\frac{1}{\sqrt{2}}^{+0.56}_{-0.73}$

Most important is the ability to distinguish different Higgs CP mixtures from one another. Referring to Fig. 1, we observe the following:[1]

- If the Higgs is the CP-even SM Higgs boson, then the pure CP-odd case is well beyond even the $\chi^2 = 36$ surface, and, in fact, it lies on roughly the $\chi^2 \sim 90$ surface, corresponding to discrimination at the 9.5σ statistical level. Even the equal CP mixture case III (the parameter location of which appears behind the $\chi^2 = 36$ surface in the figure) is ruled out at the 4.8σ level.

- If the Higgs is pure CP-odd, with SM $t\bar{t}$ coupling *magnitude*, then the CP-mixed and CP-even cases lie 17σ and 34σ away, respectively.

- If the Higgs is an equal mixture of CP-even and CP-odd, with coupling strengths specified by $a = b = c = 1/\sqrt{2}$, then the SM CP-even and pure CP-odd cases I and II are both about 6.3σ away, *i.e.* just a bit further away than the $\chi^2 = 36$ surfaces plotted.

These results improve if the e^- beam has negative polarization. The discrimination abilities are summarized in Table 2.

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[1]We refer to a parameter location on the $\chi^2 = s^2$ surface as an *s*-sigma deviation in the sense that the relative probability or likelihood compared to $s = 0$ is given by $\exp[-s^2/2]$, just as for a one-dimensional parameter space. Thus, $\chi^2 = 36$ corresponds to relative probability of 1.52×10^{-8}. This differs from the integrated probability for the parameters to lie outside the $\chi^2 = s^2$ surface, which for $\chi^2 = 36$ is 7.49×10^{-8} for 3 parameters, *i.e.* degrees of freedom.

Table 2: We tabulate the number of standard deviations, $\sqrt{\chi^2}$, at which a given input model (I, II or III) can be distinguished from the other two models, assuming $\sqrt{s} = 1\,\text{TeV}$, $m_h = 100\,\text{GeV}$, $m_t = 176\,\text{GeV}$ and $L_{\text{eff}} = 50$ fb^{-1}. Results for unpolarized beams and for 100% negative e^- polarization are given.

Input Model	Unpolarized e^- Trial Model			$P(e^-) = -1$ Trial Model		
	I	II	III	I	II	III
I	-	9.5	4.8	-	11	5.5
II	34	-	17	40	-	20
III	6.3	6.3	-	7.3	7.3	-

We can also analyze our ability to determine that the CP-violating component of $\Sigma(\phi)$, proportional to $c_5 \equiv bc$, is non-zero. We consider model III (the only one of our three models for which $bc \neq 0$). We plot the $\chi^2 = 1$ (1σ) surface in a, b, and bc space and look for the extrema of bc. We find that these extrema occur for $a \sim b \sim 1/\sqrt{2}$ and that bc can range from $-.05$ to $+.91$, assuming $L_{\text{eff}} = 50$ fb^{-1} and unpolarized beams. Clearly, we are not far from establishing a non-zero signal at the 1σ level. For twice as much luminosity, $L_{\text{eff}} \sim 100$ fb^{-1}, the extrema of bc on the 1σ surface are $+.15$ and $+.79$, and a non-zero value of bc would have been established at better than the 1σ level. At the 1σ level, $L_{\text{eff}} = 50$ fb^{-1} upper bounds on $|c_5| = |bc|$ in models I and II are 0.65 and 0.55, respectively. The above results are all somewhat better than obtained for these same models using either of the observables ($\mathcal{O}$ or $\mathcal{O}_{\text{ropt}}$) employed in Ref. [6].

4 Final Remarks and Conclusions

In this letter, we have outlined the optimal technique for extracting the coefficients that appear in a general cross section which is a sum of model-dependent coefficients times known kinematical functions. Application of this technique to $e^+e^- \to t\bar{t}h$ results in good prospects for pinning down the CP nature of the h at a $1\,\text{TeV}$ e^+e^- collider operating at an expected luminosity of $L = 200$ fb^{-1}, provided only that the h has a reasonable production cross section (roughly $\gtrsim 0.5$ fb) and that the $t\bar{t}h$ final state can be reconstructed with reasonable efficiency (roughly $\epsilon \gtrsim 0.2$). In particular, for a Higgs mass of 100 GeV and unpolarized beams, it will be possible to demonstrate, using $e^+e^- \to t\bar{t}h$ production only, that a SM Higgs boson has the expected CP-even couplings to $t\bar{t}$ and ZZ within $^{+4.3\%}_{-6.6\%}$ and $^{+51\%}_{-82\%}$, respectively, and that any CP-odd coupling to $t\bar{t}$ is less than 76% of the CP-even SM strength. The precision with which both the CP-odd and CP-even $t\bar{t}$ Higgs couplings can be determined is somewhat improved for a negatively polar-

ized electron beam, assuming there is no loss of luminosity. Most importantly, the coefficients of the various terms in the $e^+e^- \to t\bar{t}h$ cross section can be determined well enough that Higgs CP mixtures that are significantly different from one another can generally be distinguished at a substantial (sometimes very substantial) level of statistical significance.

We have implicitly assumed that the systematic error in the overall normalization of the $t\bar{t}h$ cross section will be relatively small, *e.g.* $\lesssim \pm 5\%$. If this is not the case, then one can focus on the ratios of the different cross section coefficients to one another. Our technique is easily adapted to this situation.

5   Acknowledgements

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